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Financing Innovative Green Projects with Asymmetric

Information and Costly Public Funds

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Financing innovative green projects with asymmetric information and costly public funds

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Abstract

The energy transition requires the deployment of significant programs in research and development. In absence of a long term commitment by governments on an international price of carbon various forms of national subsidies have been used. This paper analyzes the potential benefit of using subsidies conditional on success or failure of an R&D program, rather than a flat subsidy. The relationship between the state and the firm is formalized in the principal agent framework. Three potential sources of inefficiency are identified: conditions of observability of the outcome of the project, adverse selection regarding the probability of success and moral hazard. We shall show how subsidies that reward failure and subsidies that reward success mitigate these respective sources of inefficiency in a superior way as compared to flat subsidies. The gap between our second best policies and the first best is also identified. We bring together our analytical results and offer some guidance for the design of contractual investment programs such as the contractual instruments used in the Investment Program for the Future (Programme d'Investissements d'Avenir) launched in France in 2010 to promote R&D for the energy transition over the period 2010-2020.

JEL Classification:

Keywords: green innovation, financing, public support, asymmetric information.

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1 Introduction

The energy transition requires the deployment of significant programs in research and development. The success of these programs remains largely uncertain and will only be seen in tens of years. The cost benefit analysis at that horizon need combining private and social benefits. Various policies involving different forms of subsidies have been used to promote such programs. These second best policies can be justified by the absence of a (very) long-term commitment by governments on an international price of carbon to internalize the climate externality, and possibly by other externalities such as spill-overs and learning-by-doing.

However the policies actually implemented typically exhibit some ad hoc features. The gray literature has emphasized a number of pitfalls. For instance the allocation of subsidies in the Clean Development Mechanisms is based on a counter-factual that defines baseline emissions.[1] This opens the room for financing projects that would have been deployed anyway (Gillenwater and Seres; 2011; Greiner and Michaelowa; 2003). Another example concerns the promotion of renewable energy such as solar. Governments have been late in recognizing the decline in costs so that many projects also benefited from windfall profits (Brown; 2013). The REDD program has also been critically examined in this respect (Pirard; 2008).[2] More recently, in order to finance the energetic transition under tight governmental budget constraint, Aglietta et al. (2015) have proposed a scheme based on government-backed loans that otherwise would not satisfy the regulatory rules imposed on the financial capital market, again opening the room for windfall profits.

This paper formalizes such situations using the principal agent framework (Laffont and Martimort; 2002), the principal being the agency acting on behalf of the state and the agent being the firm which carries over the project. The firm invest in a project that may succeed or failed, the probability of success depends upon the type of the project and the effort of the firm. The type is known by the firm but not by the agency and the effort is non

[1] The Clean Development Mechanism is a flexible mechanism in the Kyoto protocol, that allows covered (Annex I) countries to satisfy part of their abatement objective by investing in low-carbon projects (e.g. renewable electricity) in uncovered (non-Annex I) countries.

[2] The REDD (Reducing Emissions from Deforestation and Forest Degradation), or REDD+, program allows to monitor and evaluate mitigation benefits from forest conservation in developing countries.

contractible. The agency can propose a couple of non-negative subsidies in case of success or failure.

We investigate the benefits of various forms of subsidies under asymmetric information. Our framework allows to consider three sources of inefficiency. The first one concerns observability conditions: we shall allow various conditions for the observability of the success or failure of the project. The second source concerns the presence of adverse selection: uncertainty on the success may arise from a technical risk about which the firm may be much better informed than the agency. Finally we introduce the possibility of managerial risks: the firm may have some private interest for investment which are not relevant for the agency such as spill overs for other R&D projects. This would create an opportunity for moral hazard.

From a welfare point of view one needs to balance a selection bias (induce investments in projects in as much as they are socially valuable) with a risk for windfall profits (allocate funds to projects that would have been undertaken anyway) while at the same time getting the highest possible benefit for the funds allocated by the agency. We show how flat subsidies, subsidies that reward failure and subsidies that reward success may be used to mitigate these respective sources of inefficiency. Indeed we identify benchmark models which clarify under what circumstances rewarding failure or success appear as a good incentive instrument. The first type of instrument will appear more suitable when adverse selection prevails, while the second type of instruments will be more profitable when moral hazard prevails. The gap associated with these second best instruments relative to the first best will alos be identified.

This paper has been indirectly motivated by the program monitored in France by the state agency ADEME known as the Investment Program for the Future ("Programme d'Investissements d'Avenir" to be denoted as PIA).[3] The PIA was launched in 2010 for a period of 10 years. Yearly ADEME opens calls for projects on some predefined areas. Each project is examined on its own merit, a selection is made. Then ADEME proposes a contract to each eligible project and the firm accepts or rejects the contract. Over 2010-2015 ADEME has financed more than 250 projects in various areas such as renewable energy, zero emission vehicles, green chemistry, etc. Initially the funds were allocated through flat subsidies. As from 2012 the contractual arrangements evolved to incorporate repayable advances: part of the funds allocated to a project would be paid back in case of success.

[3] http://www.ademe.fr/en/investments-for-the-future

In the concluding part of the paper we shall bring our analytical results together and suggest some guidance for the design of contractual schemes such as PIA.

Litterature review TBC

Several articles deal with the issue of financing green project under asymmetric information. Fischer (2005) provides an insightful analysis of CDM design. In her article the asymmetry of information between the agency that designs the contract and the firm which deploys the project plays a major role. It explains the potentially large windfall profit which may occur.

From a more theoretical perspective, our analyzes lies at the intersection of several strands of the literature: optimal second best taxation with externalities, and mechanism design with both adverse selection and moral hazard.[4] From this perspective, our model features both adverse selection and moral hazard, with a risk-neutral principal and a risk neutral agent, and constrained incentive schemes. The principal (the agency) is constrained to propose a single couple of non-negative subsidies. Indeed, a key difference between our analyze and the mechanism design literature is that we do not study the optimal menu offered by the principal, but mostly restrict our attention to a single couple of subsidy. We still compare numerically the optimal simple scheme with an optimal unconstrained menu.

Lewis and Sappington (2000a,b) considered mixed models with wealth constrained agents (see Quérou et al.; 2015, for a recent contribution). Laffont (1995) and Hiriart et al. (2004) analyze the regulation of environmental risk under limited liability. Interestingly, the optimal design of students loan studied by Gary-Bobo and Trannoy (2015) exhibits some features similar to our results: the students are ask to reimburse their loans in case of success but not failure.[5]

Ollier and Thomas (2013) introduces ex-post participation constraint (the firm should recover its cost even if the project fails) in a mixed model relatively similar to ours.[6] They notably show that because of countervailing incentives pooling is optimal and the principal should only reward success. Which is the case in our setting when moral hazard issues dominate, that is, when one type is much more probable than the other. Otherwise, with the

[4]Mixed models are covered in chapter 7 in Laffont and Martimort (2002).

[5]The model of Gary-Bobo and Trannoy (2015) also introduce risk aversion so that an insurance motive is added to the efficiency-rent extraction trade-off.

[6]In subsection 5.2. they replace the ex-post participation constraint by a limited liability constraint, making their model closer to ours.

present model thee are situations in which both subsidies are used or only a reward in case of failure. The key difference is the absence of a fixed cost in Ollier and Thomas (2013) which limits adverse selection issues: there is no need to finance low profitability projects but only motivate efforts.

The literature on optimal taxation and externalities (Sandmo; 1975; Bovenberg and de Mooij; 1994; Cremer et al.; 1998; Cremer and Gahvari; 2001) considers modified Pigouvian rules in second best setting, whether à la Ramsey or Mirrless. Depending on the instruments available and constraints considered (notably on the shape of the income tax) the optimal tax differs from the Pigouvian one. The fact that in our setting the optimal scheme does not consist in setting a reward equal to the marginal positive externality could be interpreted as the result of a second best situation in which public funds are costly and taxation incomplete, the profit of the successful firm cannot be taxed because of the constrained on the instruments.

The rest of the paper is organized as follows: in Section 2 we consider two benchmark models of adverse selection and moral hazard. In Section 3, we consider a situation in which both phenomenon are at play. In Section 4, hind-sights are discussed through illustrative examples. Section 5 concludes.

2 Benchmark models

Consider the following situation. A given project may or may not be deployed by a firm (the agent). Such a project is characterized by: (i) a fixed cost F, (ii) a probability of success p and a probability of failure $1-p$, (iii) a private revenue R and an external benefit b in case of success and neither private revenue nor external benefit in case of failure. If a project is not deployed the reference payoffs are zero and no fixed cost is incurred.

The regulatory agency (the principal, denoted henceforth as the agency) has selected this project and knows its characteristics F, R and b. What it knows about p will be specified in each of the following subsections. For each selected project it can propose a subsidy according to some constraints to be defined shortly. The subsidy is a take or leave it.

The firm seeks to maximize its private return: knowing the incentive scheme it decides to accept it or not and then to deploy or not the project. The regulatory agency seeks to maximize the external benefit at the minimal cost. The agency observes whether the project is or is not deployed, if it is a success or a failure as well as the associated private revenue and external benefit.

The subsidy, if any, may only depend on the outcome of a project, i.e. whether it is a success or a failure. The subsidy is s_1 in case of success and s_2 in case of failure. It is equivalent to a scheme by which the agency lends an amount s and asks for a reimbursement $s_1 - s$ in case of success and $s_2 - s$ in case of failure. We shall refer to $S^{SB} = \mathbb{R}^+ \times \mathbb{R}^+$ as the constrained class of (incentive) schemes (s_1, s_2), this defines our second best approach. $S = \mathbb{R} \times \mathbb{R}$ refers to the unconstrained class of schemes. For the sake of comparison we shall also identify the first best solution and clarify whether or not the non negativity constraints are the limiting factors in not obtaining the first best.

2.1 Rewarding failure: adverse selection

In this section we assume that the agency does not know the ex-ante probability of success p of a given project. This probability of success is distributed according to a continuous density function $g(p)$, and a cumulative distribution $G(p)$ with $G'(p) = g(p)$, defined for $p \in [0, 1]$.

The incentive scheme operates a selection under asymmetric information in which p is the type of the firm. We will show that the first best is achieved for schemes in S, there is no need to introduce a menu of contracts. The optimal second best scheme consists in rewarding failure and it does not get the first best.

The profit of a firm of type p if it deploys its project is

$$\pi(p, s_1, s_2) = p(R + s_1) + (1 - p)s_2 - F \tag{1}$$

And the expected profit is:

$$\Pi(s_1, s_2) = \int_0^1 \max\{\pi(p, s_1, s_2), 0\} dG(p) \tag{2}$$

Define $\Pi^{BAU} = \Pi(0, 0) = \int_0^1 \max\{(pR - F, 0\} dG(p)$ as the expected business as usual profit. We introduce the following assumption.

Assumption 1 *Some projects are profitable without subsidies:* $\Pi^{BAU} > 0$.

Given a scheme (s_1, s_2) there is a threshold type $\tilde{p}$ at which expected profit is null $\pi(\tilde{p}, s_1, s_2) = 0$:

$$\tilde{p}(s_1, s_2) = \frac{F - s_2}{R + s_1 - s_2} \tag{3}$$

If $R + s_1 > s_2$, as will be the case at relevant schemes, then success is more profitable than failure and all $p \geq \tilde{p}$ will be deployed.

The agency seeks to maximize the external benefit net of the subsidy that is:

$$V(s_1, s_2) = \int_{\tilde{p}(s_1,s_2)}^{1} [p(b - s_1) - (1 - p)s_2]dG(p) \tag{4}$$

Welfare is defined as the sum of the surplus of the agency (the external benefit minus the subsidy) plus the expected profit of the firm (private benefit plus subsidy minus cost). It can be written as a function of the threshold probability :

$$W(\tilde{p}) = V + \int_{0}^{1} \max\{\pi(p, s_1, s_2), 0\}dG(p) = \int_{\tilde{p}}^{1} [p(R + b) - F]dG(p). \tag{5}$$

Let us define p^{FB} the threshold type that maximizes welfare it is

$$p^{FB} = \frac{F}{R + b}, \tag{6}$$

Let W^{FB} stands for this maximum. Define p^{BAU} as the threshold type without any subsidy:

$$p^{BAU} = \tilde{p}(0, 0) = \frac{F}{R} \tag{7}$$

It is illuminating to decompose the problem of the agency in two steps.

Step 1, given a targeted threshold probability p^t the agency minimizes the expected cost of the subsidy:

$$C(p^t) = \min_{s_1,s_2} \int_{p^t}^{1} \max\{ps_1 + (1 - p)s_2, 0\}dG(p), \ s.t. \ \tilde{p}(s_1, s_2) = p^t.$$

If subsidy can be negative, then firms might be better off investing without subscribing to the scheme, and they do so if the expected subsidy is negative. This possibility explains the maximum function in the integrand. Indeed, if the subsidies are constrained to be non negative then an investing firm subscribes to the scheme. This gives the following lemma.

Step 2, the optimal choice of p^t maximizes $V = b \int_{p^t}^{1} pg(p)dp - C(p^t)$. This gives the proposition that follows.

Lemma 1 *Whatever the targeted threshold type p^t, the scheme that minimizes the expected cost of the subsidy is:*

- *For the class S, $s_2 = F$ and $s_1 = F - R + \epsilon$ with ϵ infinitely small. Then, the profits of firms that subscribe to the subsidy is null and the surplus of the agency is equal to first best welfare minus the BAU profit: $W^{FB} - \Pi(0,0)$.*

- *For the class S^{SB}, $s_1 = 0$ and $s_2 = (F - p^t R)/(1 - p^t)$. The profit of a firm of type $p \in (p^t, 1]$ is positive, and the agency surplus lower than the first best welfare.*

Proof.

Consider a change of the subsidy couple that keeps p^t unchanged: $p^t ds_1 + (1 - p^t) ds_2 = 0$. For $p > p^t$ the effect of this change on the expected subsidy received by the firm of type p is: $pds_1 + (1 - p)ds_2 = (p - p^t)(ds_1 - ds_2)$ which is negative if $ds_2 > 0$. Therefore, to reduce $C(p^t)$ the agency should increase s_2 and reduce s_1. ■

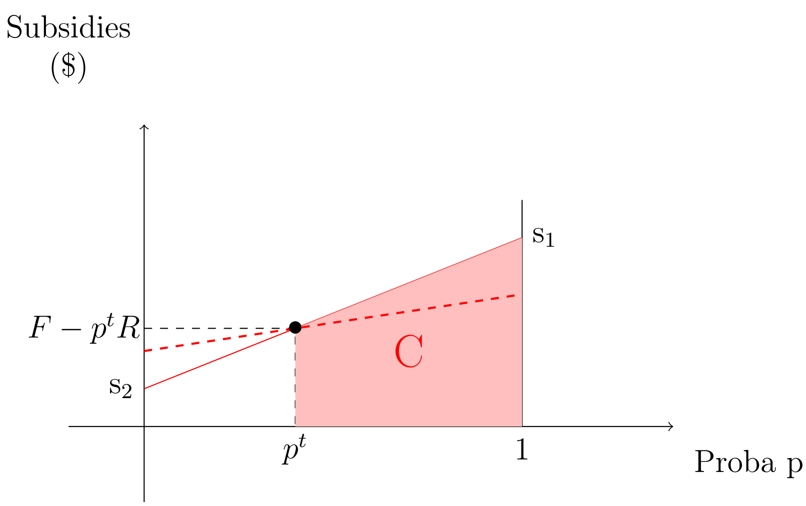


Figure 1: Expected subsidy as a function of the firm type: the red area is equal to the total expected subsidy (weighted by $g(p)$).

The result of Lemma 1 is illustrated Figure 1. Given a couple s_1, s_2 the red area corresponds to the total expected subsidy, the dashed line depicts a change of the subsidy line associated to an increase of s_2 and a reduction of s_1 that leaves the threshold firm unchanged. As can be seen such a change reduces the total expected subsidy by reducing the expected subsidy obtained by high type firms. High type firms succeed more frequently than the threshold type, they get more frequently the subsidy in case of success, and less frequently the subsidy in case of failure, the expected subsidy is then reduced by rewarding more failure and less success. At the extreme it is optimal to reward only failure in order to limit windfall profit.

We shall now show that without positivity constraints the optimal value of p^t is p^{FB} and the first best is achieved, while $p^{FB} \leq p^t \leq p^{BAU}$ with constraints. Let us denote p^{SB} the optimal value of $p^t(s_1, s_2)$ in the second best approach. Indeed the following proposition holds:

Proposition 1 *At the optimal scheme*

- *For the class S, the optimal scheme is such that $p^t = p^{FB}$ and the first best is achieved. The profit of firms that subscribe to the scheme is null, and the agency surplus is equal to W^{FB}.*
- *For the class S^{SB}, the first best is not achieved, the optimal scheme rewards failure only with $s_1 = 0$ and $s_2 \geq 0$ is such that:*

  (i) $s_2 = 0$ *and* $p^{SB} = p^{BAU}$ *if*

$$b \leq \frac{R^3}{F(R-F)} \int_{F/R}^{1} (1-p)g(p)dp \tag{8}$$

  (ii) otherwise $s_2 > 0$ *and* $p^{FB} \leq p^{SB} \leq p^{BAU}$ *with* p^{SB} *defined by the following implicit equation:*

$$p^{SB} = p^{FB} + \frac{1}{g(p^{SB})} \frac{R-F}{b+R} \int_{p^{SB}}^{1} \frac{1-p}{(1-p^{SB})^2} dG \tag{9}$$

Proof. The threshold probability as a function of s_2 is $\tilde{p}(0, s_2)$, the derivative of welfare is:

$$-[\tilde{p}b - (1-\tilde{p})s_2]g(\tilde{p})\frac{\partial \tilde{p}}{\partial s_2} - \int_{\tilde{p}}^{1} (1-p)g(p)dp \tag{10}$$

the first term is the benefit from the marginal project, the second term is the increased subsidy to all more profitable projects. the derivative of the threshold probability is

$$\frac{\partial\tilde{p}}{\partial s_2} = \frac{1-\tilde{p}}{R-s_2} = \frac{(1-\tilde{p})^2}{R-F}$$

the derivative of welfare could then be rewritten:

$$[\tilde{p}(R+b)-F]g(\tilde{p})\frac{(1-\tilde{p})^2}{R-F} - \int_{\tilde{p}}^{1}(1-p)g(p)dp \tag{11}$$

At $s_2 = 0$ $\tilde{p} = F/R$, and the derivative of welfare is negative if

$$[F(R+b)-FR]g(F/R)\frac{1}{R}\frac{(1-F/R)^2}{R-F} \leq \int_{\tilde{p}}^{1}(1-p)g(p)dp$$

point (i) follows. Otherwise, the optimal subsidy cancels the derivative of welfare and point (ii) describes the first order condition.

■

The proposition may be interpreted as follows. Consider an incentive scheme and assume that $p^{FB} \leq \tilde{p} \leq p^{BAU}$. On the one hand projects of type p such that $p^{FB} \leq p \leq \tilde{p}$ will not be implemented while they should from a first best point of view. This generates a relative loss, to be denoted as a selection bias: $W^{FB} - W(\tilde{p})$. On the other hand projects of type p such that $\tilde{p} < p \leq 1$ will be implemented but with a windfall profit, which is a second loss for the regulator: $\Pi(s_1, s_2) - \Pi^{BAU}$. This gives the following result.

Corollary 2 *The optimal second best solution minimizes the sum of the selection bias and the windfall profit.*

$$V(s_1,s_2) = W^{FB} - \Pi^{BAU} - \Big[\underbrace{W^{FB} - W(\tilde{p})}_{\textit{selection bias}} + \underbrace{\Pi(s_1,s_2) - \Pi^{BAU}}_{\textit{windfall profits}}\Big]$$

Note that if Assumption 1 is not satisfied, i.e. $\Pi^{BAU} < 0$, the optimal scheme in S is non negative. The first best is achieved in S^{SB}.

It is relatively straightforward to establish that a menu of subsidies cannot improve the situation whenever Assumption 1 holds. Whatever the initial subsidy couple proposed (s_1, s_2), there is no room for maneuver: the agency cannot propose another couple (s_1', s_2') that would be both more interesting to a firm of type $p > \tilde{p}(s_1, s_2)$ and less costly to the agency. The first condition being equivalent to $ps'_1 + (1-p)s'_2 > ps_1 + (1-p)s_2$ and the second to $ps'_1 + (1-p)s'_2 < ps_1 + (1-p)s_2$. Note that the above reasoning does not rest on the positivity constraints but on the risk neutrality of the principal and the agent, or the absence of moral hazard, which is analyzed in the following two sections.

Let us now consider as a direct extension a situation in which the agency observes with some noise whether the project is successful or not. The agency can only condition the subsidy on the observed signal, s_1 if it observes a success and s_2 if it observes a failure. Let α_1 be the probability of observing a signal of failure if the project is a success and α_2 the probability of observing a signal of failure if the project fails. We assume that $\alpha_2 \geq \alpha_1$, a perfect signal corresponds to $\alpha_2 = 1$ and $\alpha_1 = 0$ and an uninformative signal to $\alpha_2 = \alpha_1$. The subsidy obtained by a firm is $\alpha_1 s_2 + (1-\alpha_1)s_1$ in case of success and $\alpha_2 s_2 + (1-\alpha_2)s_1$ in case of failure. The threshold project is then:

$$\tilde{p}(\alpha_1 s_2 + (1-\alpha_1)s_1, \alpha_2 s_2 + (1-\alpha_2)s_1),$$

and the expected total subsidy is

$$\int_{\tilde{p}}^{1} \Big\{ p\big[(1-\alpha_1)s_1 + \alpha_1 s_2\big] + (1-p)\big[(1-\alpha_2)s_1 + \alpha_2 s_2\big] \Big\} dG(p)$$

Corollary 3 *If the success and failure of a project are not perfectly observable,*

- *For the class S: the first best is achieved.*
- *For the class S^{SB}: The optimal scheme remains of the form $s_1 = 0$ and $s_2 > 0$. The second best threshold type, the expected subsidy, the agency surplus, the welfare and the profit of firms only depend on the ratio α_1/α_2.*
  - *If $\alpha_1 = 0$ (success is perfectly observed), then, whatever α_2, at the optimal second best scheme, the threshold probability, welfare and $\alpha_2 s_2$ do not depend on α_2 and correspond to perfect observability situation.*
  - *Otherwise, with a homogeneous distribution, the threshold probability is higher, and, welfare and the agency surplus are lower than in the case with a perfect signal.*

Proof. see Appendix A ■

Technically the agency would like to subsidize failure and not success, whether a failure is not properly identified ($\alpha_2 < 1$) is not an issue since it can play with s_2 to increase the expected subsidy in case of failure. The agency is mainly concerned by the noise in case of success, $\alpha_1 > 0$. The ratio α_1/α_2 is the number of $ awarded to successful projects for any $ awarded for failed projects, this ratio determines the inefficiency of the subsidy scheme with noise.

Two comments are in order. Firstly, an imperfect signal may originate from a manipulation of the agent. The mere possibility of such a manipulation deteriorates the efficiency of the incentive scheme. Secondly, in case of an uninformative signal, the optimal scheme is equivalent to a flat subsidy $s_1 = s_2$ since the subsidy will be given independently of the signal received.

2.2 Rewarding success: moral hazard

In this section we introduce moral hazard. The probability of success is a function of an effort e and the type θ of a project, $p(e,\theta)$. The cost of effort is a function of e and θ, it is denoted $f(e,\theta)$. The type of the firm θ is known by the agency, but neither the effort nor its cost are observable by the agency. We shall show that without the non negativity constraints the first best is achieved. With the non negativity constraints rewarding success only is the second best solution, however the first best is not achieved.

We use the following specification:

$$p(e,\theta) = \theta + e(1-\theta) \tag{12}$$

and

$$f(e,\theta) = (1-\theta)\frac{\gamma}{2}e^2 \tag{13}$$

The probability of success $p(e,\theta)$ is a clockwise rotation of the initial probability $p(0,\theta)$ around the point ($\theta = 1$, $p = 1$).[7] This specification is

[7] This specification has the following appealing properties: Without any effort the probability of success is equal to the type (contrary to Lewis and Sappington; 2000b) which will ease the comparison with the pure adverse selection model studied above. It is less costly for a good project to ensure success by setting $e = 1$, which seems realistic. The constraint $p \leq 1$ turns into $e \leq 1$ which is independent of the type. Furthermore, this specification has the nice tractability property that the level of effort will not depend on the type.

meaningful as long as $e \leq 1$ which implies some restrictions on our parameters γ, R, b to be made precise shortly.

For a scheme (s_1, s_2) we get the agency surplus:

$$v(\theta, e, s_1, s_2) = p(e,\theta)(b - s_1) - (1 - p(e,\theta))s_2 \tag{14}$$

the profit of the firm:

$$\pi(\theta, e, s_1, s_2) = p(e,\theta)[R + s_1 - s_2] + s_2 - [F + f(e,\theta)] \tag{15}$$

and the welfare:

$$w(\theta, e) = p(e,\theta)[R + b] - (F + f(e,\theta)) \tag{16}$$

The effort that maximizes welfare is:

$$e^{FB} = min\frac{R+b}{\gamma}, 1 \tag{17}$$

Note that the effort which maximizes the profit for a given couple of subsidies depends only on $(s_1 - s_2)$

$$e(s_1, s_2) = \frac{R + s_1 - s_2}{\gamma}, \tag{18}$$

which indeed corresponds to the effort actually performed by a firm if the above expression is between 0 and 1. Note that $e^{BAU} = e(0,0) = R/\gamma$ so that $e^{BAU} \leq e^{FB}$.

We can now derive θ^{BAU} and θ^{FB} that give the respective thresholds for a firm to deploy the project without subsidy and for a first best deployment. It is easily seen that:

$$\theta^{BAU} = \frac{1}{R}\frac{2F\gamma - R^2}{2\gamma - R} \tag{19}$$

$$\theta^{FB} = max\left\{\frac{1}{R+b}\frac{2F\gamma - (R+b)^2}{2\gamma - (R+b)}, 0\right\} \tag{20}$$

The above analysis suggests to calibrate the parameters as follows:

Other cost functions can capture that less effort is optimal as the initial probability of success increases, which would have cumbersome consequences.

Assumption 4 *The first best effort is less than 1:* $\gamma > R + b$.

Assumption 5 $R^2 \leq 2F\gamma$ *and* $F \leq R$ *so that* $0 \leq \theta^{BAU} \leq 1$.

Note that $\theta^{FB} \geq 0$ if and only if $(R+b)^2 \leq 2F\gamma$. As b increases θ^{FB} decreases from θ^{BAU} to 0. BAU is such that for $\theta \leq \theta^{BAU}$ the firm does not deploy the project, and for $\theta^{BAU} < \theta \leq 1$, it deploys the project and makes the effort e^{BAU}.

We now describe the optimal second best scheme $(s_1, s_2) \in S^{SB}$ as a function of the type θ of the project. The agency should decide whether to ensure the deployment of a project for $\theta \leq \theta^{BAU}$, and whether to further motivate effort. Firstly, if the agency ensures the deployment of a project it is optimal to do so by rewarding success and not failure i.e. $s_1 > 0$ and $s_2 = 0$ since it maximizes the effort of the firm. Secondly, if the agency subsidizes a project it has to decide whether to solely ensure the deployment, leaving no rent to the firm, or further subsidizing success to increase the firm's effort. The occurrence of these two possibilities depends on the value of the parameters b, γ, R and F.

For small values of the external benefit b, we have $\theta^{FB} \geq 0$ and the agency does not subsidize the firm as long as $\theta \leq \theta^{FB}$. For a higher type θ, the agency subsidizes the project and calibrates the subsidy so that the firm's profit is null. For larger values of b, the agency might let a windfall profit to the firm to achieve a high probability of success. The following proposition makes this precise and proves that there will be a windfall profit as soon as $R + b \geq 2\sqrt{2F\gamma}$.

Define:

$$s_{1A}(\theta) = \frac{b-R}{2} - \gamma\frac{\theta}{1-\theta} \tag{21}$$

and

$$s_{1B}(\theta) = \gamma\frac{\theta}{1-\theta}\left[\sqrt{1 + \frac{2F}{\gamma}\frac{1-\theta}{\theta^2}} - 1\right] - R \tag{22}$$

Proposition 2 *At the optimal second best scheme* (s_1^*, s_2^*) *in* S^{SB}*, the subsidy in case of failure is null:* $s_2^* = 0$*. The precise expression of the subsidy in case of success* $s_1^*(\theta)$ *depends on the two following cases:*
Case 1: If $R + b \leq 2\sqrt{2F\gamma}$ *then*
- *if* $\theta \leq \theta^{FB}$ *the optimal subsidy is null, the project is not implemented;*

- if $\theta^{FB} \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is implemented and the firm gets no windfall profit;

- and if $\theta \geq \theta^{BAU}$ the optimal subsidy is null, it is business as usual, the project is implemented and the firm gets no windfall profit.

Case 2: if $R + b \geq 2\sqrt{2F\gamma}$ there is a threshold θ_A such that:

- if $\theta \leq \theta_A$ the optimal subsidy is $s_{1A}(\theta)$, the project is implemented and the firm gets a windfall profit;

- if $\theta_A \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is implemented and the firm gets no windfall profit;

- and if $\theta \geq \max\{\theta_A, \theta^{BAU}\}$ the optimal subsidy is null, it is business as usual, the project is implemented and the firm gets no windfall profit;

Proof in Appendix B

The precise expressions of θ_A cannot be determined explicitly, it is the type at which the expression s_{1A} is either equal to s_{1B} or null. Above this type θ_A, it is not worth conceding a rent to the firm in order to increase effort. Depending on the value of b this threshold is either larger or lower than θ^{BAU}.

With an unconstrained scheme, the agency implements the first best, welfare is then equal to $w(\theta, e^{FB})$, and the firm gets its BAU profit. The proof of this lemma is straightforward.

Lemma 2 *The first best is obtained with a scheme in S such that:*

- if $\theta \leq \theta^{FB}$, no subsidy is proposed and the project is not deployed,

- if $\theta \geq \theta^{FB}$, the optimal scheme is such that $s_1 - s_2 = b$ and s_2 such that $\pi = \pi^{BAU}$.

Welfare is then $w = p(e, \theta)(b - (s_1 - s_2)) - s_2 = -s_2$, the subsidy s_2 is negative and corresponds to a tax on profit.

3 The general model with adverse selection and moral hazard

We now investigate the more general case in which the firm can make an effort and knows its type θ but the agency does not. We keep the same specification for the effort and the cost. We have two questions in mind. Firstly since there is some contradiction between rewarding success and rewarding failure we wonder whether there are situations in which both subsidies could be strictly positive. Secondly we want to know whether the introduction of a menu of contracts is necessary and sufficient to achieve the first best.

The fully general model with a continuum of type proved difficult to solve. So, we consider only two types: θ_L and θ_H with $\theta_L < \theta_H$. The probability of type θ_H is denoted λ. We analyze the influence of the distribution of types. We shall show that there is a range for λ for which both subsidies are strictly positive while for lower λ rewarding success prevails and for high λ rewarding failure prevails. We also show that there are two potential benefits to using a menu in S, i.e. inducing different effort levels depending on the type of the firm and taxing profits. Still the optimal menu leaves a gap as compared with the first best: asymmetry of information generates some inefficiency independently of constraints on the incentive schemes.

The following assumption is introduced to get the results.

Assumption 6 *We take $(R+b) < 2\sqrt{2F\gamma}$ and θ_L and θ_H such that $\theta^{FB} < \theta_L < \theta^{BAU}$ and $\theta^{BAU} < \theta_H$.*

To get intuition on the structure of the optimal second best scheme start with a situation in which the cost of effort (γ) is very high. We have an adverse selection situation in which for low values of λ it is worthwhile to induce a low type firm to deploy the project through rewarding failure (and make no profit) at the expense of allowing a rent to a high type firm. For a high enough value of λ the agency should not give any subsidy, the high type firm uses its BAU strategy while a low type firm does not deploy. As γ decreases, for low values of λ, it may become worthwhile to induce a low type firm to make an effort through rewarding success, the incremental rent for high type firm being more than compensated. How do these two situations of rewarding success and rewarding failure combine together? As λ increases the balance between the benefit accruing from a higher effort from a low type firm should exactly balance the increase in the rent of the high type firm. The following lemma precisely defines the relationship between s_1 and s_2 for these intermediary situation.

Lemma 3 *At the optimal scheme (s_1^*, s_2^*) in S^{SB}, if both subsidies are strictly positive then they satisfy:*

$$s_1^* - s_2^* = b - \frac{\gamma\lambda(\theta_H - \theta_L)}{(1-\theta_L) - 2\lambda(\theta_H - \theta_L)}(1 - e^{FB}) \tag{23}$$

and s_2^* is such that the profit of the low type firms is null, it solves:

$$s_2^* = F - \theta_L(R + (s_1^* - s_2^*)) - (1 - \theta_L)\frac{(R + (s_1^* - s_2^*))^2}{2\gamma} \tag{24}$$

The proof is in Appendix C.

We now characterize the optimal second best scheme for all values of λ.

Proposition 3 *The optimal scheme s_1^*, s_2^* in S^{SB} depends on three thresholds λ_1, λ_2 and λ_3 as follows:*
– for $0 < \lambda \leq \lambda_1 :$ $s_1^* > 0$ and $s_2^* = 0; s_1^* = s_{1B}(\theta_L)$ given by equation (22)
– for $\lambda_1 < \lambda < \lambda_2 :$ $s_1^* > 0$ and $s_2^* > 0$ given by Lemma 3 ;
– for $\lambda_2 < \lambda < \lambda_3 :$ $s_1^* = 0$ and $s_2^* > 0$ such that $\pi(\theta_L, e, 0, s_2^*) = 0 :$

$$s_2^* = R - \gamma + \gamma\left[1 - \frac{2}{\gamma}\frac{R - F}{1 - \theta_L}\right]^{1/2}$$

– for $\lambda_3 < \lambda \leq 1 :$ $s_1^* = 0$ and $s_2^* = 0.$

The profit of a L firm is always null, a H firm gets a windfall profit as long as $\lambda < \lambda_3$.

When λ is low, Proposition 2 is approximately true and a low type firm receives a subsidy in case of success and no rent. This subsidy both promotes deployment and effort by a low type firm. As λ increases, an growing part of the expected subsidy is diverted by a high type H firm through a rent. The agency then progressively shifts the subsidy from success to failure to reduce the expected cost of the subsidy, it comes at the expense of a reduced effort for both types. At some point only failure is subsidized, the effort is then low but the informational rent minimized. For a very large λ, no subsidy for failure is provided, and only a high type deploys the project. This structure of the incentive scheme illustrates how the two conflicting goals of rewarding success and rewarding failure interact as a function of λ.

We now show that even with no constraints and a menu of contracts the agency cannot implement the first best. because of both the non-negativity constraint on subsidies and asymmetric information. The optimal scheme with asymmetric information and unconstrained subsidy is a relatively standard mechanism design problem: the agency (the principal) should propose a menu of couples $\{(s_{1L}, s_{2L}), (s_{1H}, s_{2H})\}$ to the firm which self selects. The menu is designed so that a type $i = L, H$ chooses the item (s_{1i}, s_{2i}). Several cases can arise whether only a high type or both types deploy their project, and whether a low type exerts an effort.

For the sake of simplicity, and to focus on the role of asymmetric information, contrary to previous sections we assume that the agency can capture the rent from a high type.[8]

Assumption 7 *A firm cannot deploy a project without the consent of the agency.*

Furthermore, we assume that a low type project is worth implementing, from a welfare perspective, even without effort. This assumption is satisfied in our numerical illustration. With this assumption it is always beneficial to encourage the low type firm to invest.

Assumption 8 *The low type is such that $\theta_L(R+b) - F > 0$.*

With these two additional assumptions only two situations can arise. In both situations a low type project is deployed with a sub-optimal effort, possibly null, and a high type exerts an optimal effort. In one case a high type gets a rent and the low type exerts an effort; in the other case, a high type gets no rent and a low type exerts no effort.

Proposition 4 *If the agency can propose a menu $\{(s_{1L}, s_{2L}), (s_{1H}, s_{2H})\} \in S^2$, under Assumptions 7 and 8, there is a threshold λ_{menu} that delineates two cases.*

In both cases, the high type exerts the first best effort: $s_{1H} - s_{2H} = b$, and the low type gets a null profit. Furthermore:

- *For $\lambda > \lambda_{menu}$:*
  - *The low type exerts no effort: $s_{1L} - s_{2L} = -R$*
  - *Both the low and high type gets no profit: $s_{2L} = F$ and*

$$s_{2H} = F - \theta_H(R+b) - (1-\theta_H)\frac{(R+b)^2}{2\gamma} < 0$$

[8] Without this assumption, the high type would compare its profit with (s_{1H}, s_{2H}) with both its profit with (s_{1L}, s_{2L}) and its BAU profit. The menu would then have to satisfy the self selection constraint and this new participation constraint. The high type would still be incentivize to make the first best effort ($s_{1H} - s_{2H} = b$), but it would get at least its BAU profit. The scheme offered to a low type would be more attractive than BAU for a high type for a low probability of a high type, but not for a sufficiently large probability of a high type. In that case, the situation would be close to Lemma 2.

- *For $\lambda < \lambda_{menu}$:*

  - The low type exerts a suboptimal effort with

$$s_{1L} - s_{2L} = b - \frac{\lambda(\theta_H - \theta_L)}{(1-\lambda)(1-\theta_L) - \lambda(\theta_H - \theta_L)} [\gamma - R - b] \tag{25}$$

  - The high type gets a positive informational rent.

The proof is in Appendix C.3. When a high type is highly probable, the agency uses a couple $s_{2L} = F$ and $s_{1L} = F - R < 0$ to trigger the deployment of a low type project (thanks to Assumption 7) without generating a rent for a high type. It is then possible to complete the menu with a couple that extract the maximimum surplus from a high type firm. It is not worth encouraging effort of a low type because of the rent to a high type it would generate. When a high type is less probable, the situation is standard: the high type exerts the optimal effort and gets an informational rent, the low type exerts a suboptimal effort without getting any profit. The trade-off implicit in the expression of $s_{1L} - s_{2L}$ displayed is the following: reducing s_{1L} necessitates to compensate a low type by increasing s_{2H}, it reduces the effort of a low type, and it allows to reduce the rent of the high type. The last rent reduction effect is reflected in the difference $\theta_H - \theta_L$: a high type being more probably successful than a low type, re-equilibrating the subsidy from success (s_{1L}) to failure (s_{2H}) reduces the windfall profit of the high type.

As the frequency of a high type decreases, the difference $s_{1L} - s_{2L}$ increases and converges toward b, the rent of a high type increases. The similar pattern as in Proposition 3 emerges for the scheme offered to low type: when a high type is less probable, success is rewarded to encourage effort; when a high type is more frequent, success is less rewarded (even penalized), the subsidy is shifted to failure in order to limit the rent captured by high types.

4 Numerical illustrations and discussion

In all our numerical examples we take $F = 1$ (which may be seen as a normalization) and $R = 1.5$, assumption 1 is satisfied. The firm profit in a business as usual situation would be positive if the probability of success is higher than $p^f = F/R = .67$.

4.1 Adverse selection: the superiority of rewarding failure over a flat subsidy

We first illustrate how rewarding failure optimizes between the selection bias and the windfall profit and show how a flat subsidy would be much less effective in this optimization. We consider the case of an homogeneous distribution ($g(p) = 1$ and $G(p) = p$). Then the optimal second best solution and the optimal flat subsidy (noted with the label FS) can be easily derived.

Corollary 9 *For the class S^{SB}, two cases should be distinguished:*

- *If $b < R(R-F)/(2F)(= 0.375)$, then $s_1^* = s_2^* = 0$,*
- *otherwise the optimal threshold probability is such that*

$$p^{SB} = \frac{1}{2}\frac{R+F}{R+b} \tag{26}$$

and the optimal scheme satisfies:

$$s_1^* = 0 \text{ and } s_2^* = \frac{F(R+2b) - R^2}{R+2b-F} \tag{27}$$

And similarly we get for the optimal flat subsidy:

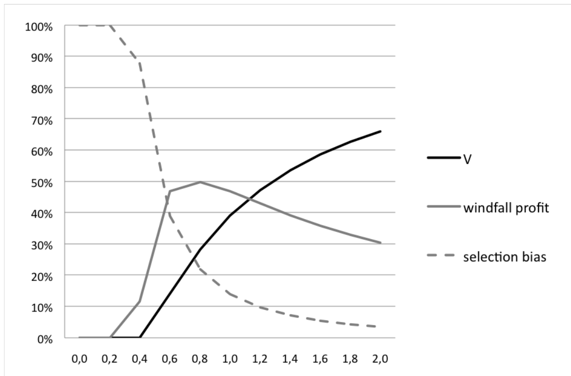
Corollary 10 *With a flat subsidy, that is $s = s_1 = s_2$, two cases should be distinguished:*

- *If $b < R(R-F)/F(= 0.75)$, then $s_1 = s_2 = 0$,*
- *otherwise the optimal threshold probability is such that*

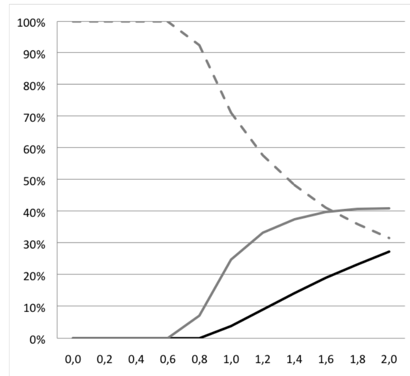
$$p^{FS} = \frac{R+F}{2R+b} \tag{28}$$

and the optimal subsidy is :

$$s_1 = s_2 = \frac{F(R+b) - R^2}{2R+b} \tag{29}$$



(a) Optimal second best solution



(b) Optimal flat subsidy

Figure 2: Decomposition of welfare as a share of W^{FB} as b increases

The relative efficiencies of both schemes and the way they optimize between the selection bias and the windfall profit are depicted in Figure 2a and 2b. The superiority of rewarding failure appears clearly. For instance for $b = 1$ rewarding failure allows for getting 39% of the first best net welfare gain defined as $W^{FB} - \Pi(0, 0)$ while this drops to only 4% with a flat subsidy. Note also that the social benefit b need to be higher than some threshold (and a much higher one in case of flat subsidy) to induce the agency to subsidize a project. This comparison indirectly sets a value for the observability of success versus failure. Indeed in the case of an uninformative signal the agency would only be able to propose a flat subsidy, a much less efficient scheme.

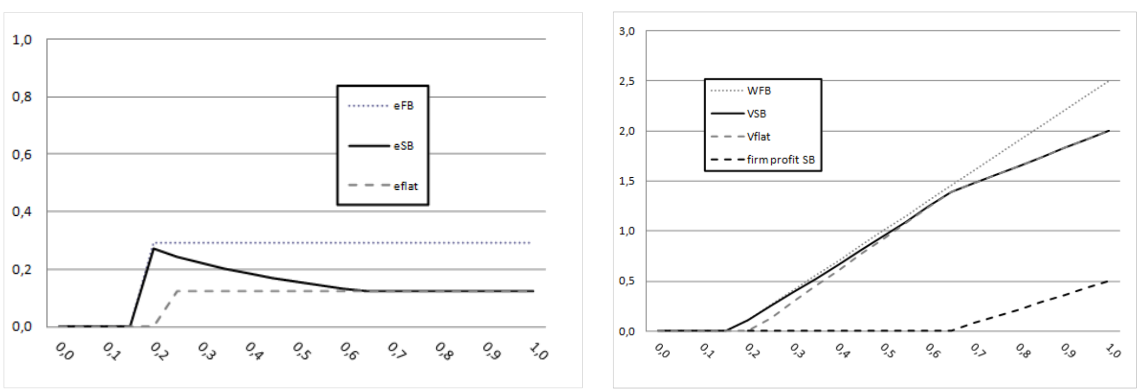
4.2 The case for rewarding success in the presence of managerial risks

Moral hazard becomes an important issue as the social benefit increases. Then the agency more and more prefers to get a success. We introduced the idea that increasing the probability of success is costly for the firm, to capture that the firm may get spill-overs from the project, which are not reflected in the observable profit R. Inducing the firm to make an effort would then mean that the firm concentrates on the project as such to the detriment of the spill-overs.

Recall the formalization of section 2.2.: the level of effort induced by a scheme (s_1, s_2) depends only on the difference $s_1 - s_2$ and not on the type of the firm θ. A flat subsidy induces no further effort than the BAU effort, so that it is worth nothing to the agency. The increase in the probability of success for a given level of effort decreases as the type of firm (the probability of success without effort) increases. Under these circumstances we may expect that the benefit of rewarding success over a flat subsidy would increase as the social benefit increases and decrease as the type of the firm increase. This is illustrated in the following graphs.

We keep $F = 1$, $R = 1.5$ and set $\gamma = 12$. We have $\theta^{BAU} = .64$. We compute the optimal second best scheme for $b = 2$ and $b = 10$ for all θ in $[0, 1]$. The limit value between cases 1 and 2 in Proposition 2 for b is such that $R + b = 2\sqrt{2F\gamma}$, that is, $b = 8.3$.

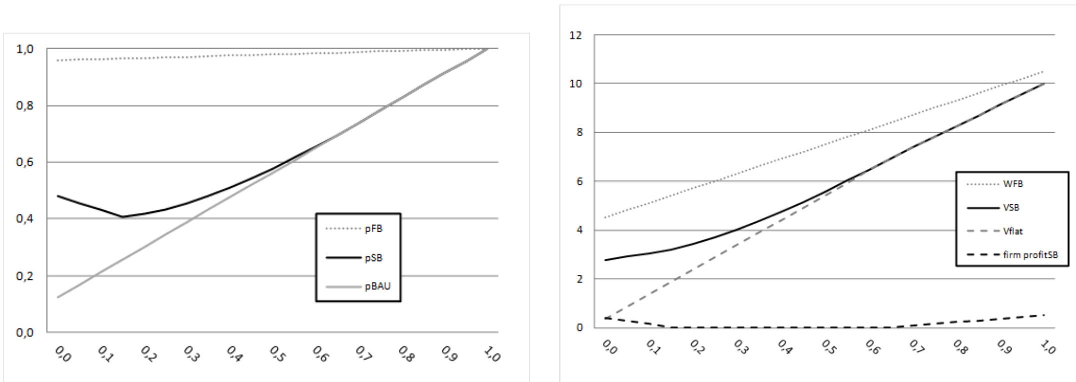
For $b = 2$, Case 1 of Proposition 2 prevails. We have $\theta^{FB} = .16$. If $\theta \leq \theta^{FB}$ the agency does not subsidize the project; if $\theta^{FB} \leq \theta \leq \theta^{BAU}$ the agency proposes s_{1B}, the project is implemented without any windfall profit to the firm, but the agency gets less than the first best welfare. If $\theta^{BAU} \leq \theta \leq 1$ the agency does not propose any subsidy, the project is implemented as BAU. Figure 3a depicts the corresponding probabilities of success, given the associated efforts, e^{BAU}, e^{SB}. Figure 3b gives the corresponding the payoffs.



(a) Effort (b) Payoffs

Figure 3: Effort (a) and payoffs (b) with moral hazard as a function of type θ of the project with $b = 2$.

Consider now case 2 with $b = 12$. We have $\theta^{FB} = 0$ and a numerical analysis shows that $\theta_A = .15$. Figures 7 and 8 are similar to Figures 4 and 6. Observe that p^{FB} is very close to 1 and the project is always deployed. The agency proposes s_{1A} if $0 \leq \theta \leq \theta_A$, s_{1B} if $\theta_A \leq \theta \leq \theta^{BAU}$ and no subsidy if $\theta^{BAU} \leq \theta \leq 1$. Note that in the first situation the firm gets a windfall profit.



(a) Probability of success

(b) Payoffs

Figure 4: Probability of success (a) and payoffs (b) with moral hazard as a function of type θ of the project with $b = 10$.

Comparing these two cases two comments can be made. Firstly, the benefit of rewarding success rather than using a flat subsidy is higher for low and medium values of θ; for high values the benefit of rewarding success is too low relative to letting the firm implement e^{flat}. Secondly, the higher b the more important for the agency to have the project implemented. So that for low values of θ the agency provides a higher reward for success, which strongly increases the probability of success, provides a benefit for the agency and a windfall profit for the firm. There is a tradeoff between encouraging success and letting a windfall profit. This clearly shows that the superiority of rewarding success over a flat subsidy depends on the magnitude of the social benefit.

4.3 Why proposing a menu may not be better than combining rewarding success and rewarding failure

The above discussion suggests that rewarding success and rewarding failure pursue two distinct objectives. If the initial probability of success is low and if there are managerial risks reward success. If there are significant asymmetry of information on the probability of success between the agency and the firm and if this opens the opportunity for windfall profit reward failure. In a situation in which these two issues co-exist designing an optimal second best scheme becomes tricky because each type of scheme generates perverse outcomes when used inappropriately. We shall illustrate this point through a numerical example and show how introducing a menu allows for solving the dilemma.

We stick to $F = 1$, $R = 1.5$, $b = 2$ and $\gamma = 12$. We have $\theta^{FB} = .16$ and $\theta^{BAU} = .64$. We introduce two discrete types: a low type for which $\theta^{FB} < \theta_L = .3 < \theta^{BAU}$ and a high type for which $\theta_H = .75 > \theta^{BAU}$. Assumption 6 is satisfied: A low type firm would not implement the project but it would be socially valuable to do it. A high type firm would implement the project without subsidy. The parameter λ denotes the probability for the firm to be of the high type.

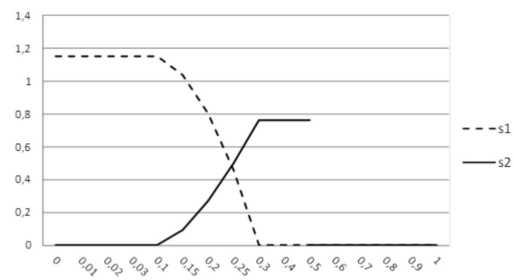
We can derive numerically the thresholds for λ to approximately be $\lambda_1 = .1$, $\lambda_2 = .3$ and $\lambda_3 = .6$. Figure 5(a) depicts the second best optimal solution: only reward success if $0 \leq \lambda \leq \lambda_1$, reward both success and failure if $\lambda_1 \leq \lambda \leq \lambda_2$, only reward failure if $\lambda_2 \leq \lambda \leq \lambda_3$, and provides no subsidies if $\lambda_3 \leq \lambda \leq 1$. The extreme cases correspond to intuition. Reward success if λ is small, a situation in which effort should be encouraged and windfall profit discounted by a low probability of occurence. Reward failure if λ is large for the reverse reasoning, up to a point at which type θ_L does not matter anymore and BAU should be prefered, letting no windfall profit to type θ_H. The interesting part is when $\lambda_1 \leq \lambda \leq \lambda_2$ for which we expect the most from a menu.

Figure 5(b) gives the optimal menu. Its construction follows from Proposition 4. Observes that it uses negative values for subsidies so as to get back the profit of the firm.[9]

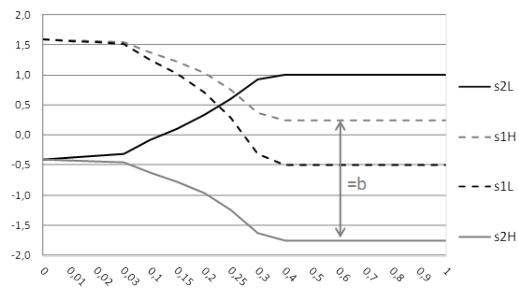
Figure 5(c) gets to the point. It allows for comparing the second best effort and the conditional menu efforts. The second best effort decreases as λ gets into the zone $\lambda_1 \leq \lambda \leq \lambda_2$. The effort for type θ_L is sacrificed for not giving a windfall profit to type θ_H. With a menu this is also the case but the first best effort for a high type firm is elicited, for a low benefit. Altogether we do not expect that the menu increases a lot the expected benefit for the agency. This argument does not carries over to large values of λ since negative subsidies allow the agency to recover the profit of the firm. As a

[9] We have not derived the optimal second best menu but we think that this would not qualitatively alter our argumentation.

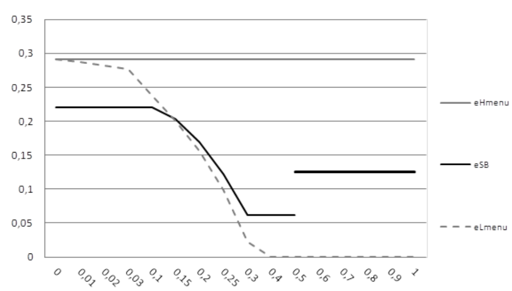
side comment observe that it is optimal not to induce the first best effort for the low type firm (a standard result of contract theory) this explains why the first best cannot be achieved with a menu independently of negativity constraints.



(a) Optimal second best scheme as a function of the probability λ of high type



(b) Optimal menu as a function of the probability λ of high type



(c) Efforts as a function of the probability λ of high type

Figure 5: Optimal second best scheme, menu and efforts with two types L and H as a function of the probability λ of a high type ($R = 1.5$, $F = 1$, $\gamma = 12$ and $b = 2$).

Figure 4.3 confirms that the benefit of using a menu is not significant for $\lambda_1 \leq \lambda \leq \lambda_2$. As expected the benefit of using a menu (without non-

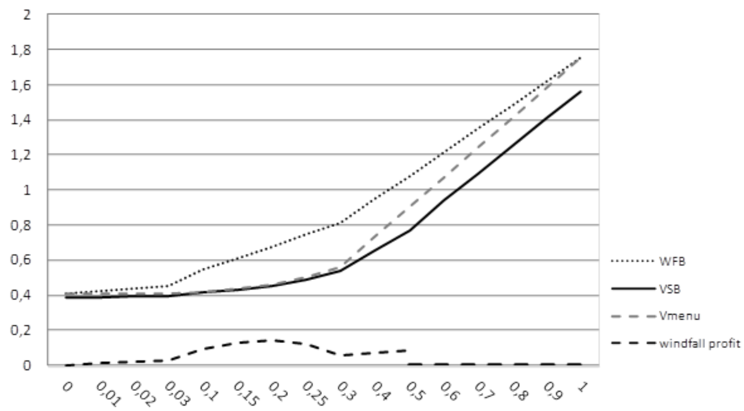


Figure 6: Payoffs as a function of the probability of a high type with First Best, Second Best and a menu (unconstrained).

negativity constraints) becomes significant for high λ through recovering the profit of the firm. In this Figure the expected windfall profit for the firm with a second best optimal scheme is also displayed (multiplied by 10 to be seen in the graph).T It exhibits two peaks reflecting the conflicting forces associated with rewarding success and rewarding failure.

5 Conclusion

This paper is concerned with public financing of R&D programs for the energy transition that have the following characteristics: the program has an uncertain outcome ranging from full success to total failure, the social benefit is associated with success which also generates private gains, public financing takes the form of subsidies, the state agency which monitors the subsidy allocation process has much less information about the economics of the project than the firm.

Our analysis formalizes such situations using a principal agent framework with constraints on the incentive structure. We identify three potential sources of inefficiency and suggest the contractual arrangements that can be implemented to mitigate them. The first source of inefficiency concerns observability conditions. We allow various situations for the observability of the success or failure of the project on top of a prerequisite to observe the reality of the investment. The second source concerns the presence of adverse selection: uncertainty on the success may arise from a technical risk about which the firm may be much better informed about than the agency. Finally we introduce the possibility of managerial risks: the firm may have some private interest for investment which are not relevant for the agency such as spill overs for other R&D projects. This would create an opportunity for moral hazard.

The relevance of our analysis can be discussed in line with the evolution of the practice of ADEME for the PIA, a context which particularly fits our model. Initially only flat subsidies were used. This generated two kind of problems. On the one hand, the identification of windfall profits for projects that would have been deployed without any subsidy and, on the other hand, the identification of projects that would be socially beneficial but could not be subsidized given a EU constraint that limits the level of subsidy at a given percentage of the investment cost. This led the agency to use repayable advances. We prove that such schemes which amounts to rewarding failure are indeed optimal when the asymmetry of information takes the form of adverse selection. Moreover repayable advances allow for relaxing the EU constraint.

The difficulty in the observation of success, including managerial risks by the firm, led the agency to formalize success through steps and have repayable advance paid back in part along the way as well as to increase in the interest rate as the repayment schedule is delayed. This may be seen as a counterpart of unobservable spill over to other projects. We show that the advantages of repayable advances decrease as observability conditions deteriorate. Moreover rewarding success is shown to be a good strategy when the asymmetry of information takes the form of moral hazard.

All this evolution at ADEME is very pragmatic, incremental and follows rules of thumb. Our economic analysis, in spite of our simplifying assumptions, may already offer some guidance for the design of the contractual arrangements relevant to each project. To be fully relevant our framework would benefit from several extensions, which may be interesting for their own sake. We can think of three extensions. Firstly a complete analysis of our general model would be helpful to understand how rewarding success and rewarding failure may complement each other in some situations. Possibly a third benchmark model could be identified along the way. Secondly we only investigate a situation in which the project leads to two extreme outcomes, failure or success, a more realistic model should allow for a continuous set of outcomes that would be imperfectly observed by the agency. Thirdly, the asymmetry of information may involve another party. It appeared that ADEME sometimes plays the role of a middle man between the firm and the banking system. Due to its technical expertise ADEME the asymmetry of information is much more acute between the firm and the banking system than from the firm and ADEME. The formalization should explicitly take this dynamic aspect into consideration and analyze how the contractual arrangement should evolve as this asymmetry reduces over time.

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Appendix

A Proof of corollary 3

Let us denote $\sigma_1 = \alpha_1 s_2 + (1 - \alpha_1)s_2$ and $\sigma_2 = \alpha_2 s_2 + (1 - \alpha_2)s_2$ the subsidy obtained in case of success and failure respectively.

For the unconstrained class S: with the couple of subsidy: $s_1 = F - \alpha_2 R/(\alpha_2 - \alpha_1)$ and $s_2 = F + (1-\alpha_2)R/(\alpha_2 - \alpha_1)$, the expected subsidies are $\sigma_1 = F$ and $\sigma_2 = F - R$ which implement the first best.

The reasoning of Lemma 1 can be reproduced: an increase of σ_2 coupled with a reduction of σ_1 that leaves $\tilde{p}$ unchanged reduces the total expected subsidy. Consequently it is optimal to set $s_1 = 0$ and $s_2 > 0$.

Then, with $s_1 = 0$, $\sigma_1 = x\sigma_2$ with $x = \alpha_1/\alpha_2$ and the threshold probability is $\tilde{p}(x\sigma_2, \sigma_2)$, the regulator surplus is

$$V(x\sigma_2, \sigma_2) = \int_{\tilde{p}}^{1} \Big[p(b - x\sigma_2) - (1-p)\sigma_2\Big] dG(p)$$

and welfare is $W(\tilde{p}(x\sigma_2, \sigma_2))$.

If $\alpha_1 = 0$ then $x = 0$ and the surplus of the regulator, the profit of firms, and total welfare could all be written as functions of σ_2 without any other dependence on α_2. The optimum second best scheme is then similar to the scheme described by Proposition 2 with $\alpha_2 s_2$ being independent of α_2.

The total derivative of the threshold type w.r.t. σ_2 is:

$$\frac{d\tilde{p}}{d\sigma_2} = -\frac{1 - (1-x)\tilde{p}}{R - (1-x)\sigma_2}$$

the first order condition satisfied at the optimal scheme is

$$\Big[\tilde{p} - p^{FB}\Big] g(\tilde{p}) \frac{1 - (1-x)\tilde{p}}{R - (1-x)\sigma_2} = \int_{\tilde{p}}^{1} \Big[px + (1-p)\Big] dG(p)$$

which could be rewritten

$$\tilde{p} = p^{FB} + \frac{1}{g(\tilde{p})} \frac{R - (1-x)F}{R + b} \int_{\tilde{p}}^{1} \frac{px + (1-p)}{\big(1 - (1-x)\tilde{p}\big)^2} dG(p)$$

with a homogeneous distribution it gives:

$$\tilde{p} = p^{FB} + \frac{R - (1-x)F}{R + b} \frac{1}{2(1-x)} \Big[1 - \frac{x^2}{\big(1 - (1-x)\tilde{p}\big)^2}\Big]$$

Let us prove that p^{SB} increases with respect to x (brutal calculations):

- the RHS side is a decreasing function of $\tilde{p}$

- It is increasing with respect to x:

  Its derivative wrt to x is

$$\frac{(1-\tilde{p})}{2(R+b)}\frac{2Fx+R[(1-\tilde{p})^2-x\tilde{p}(1+\tilde{p})]}{\big(1-(1-x)\tilde{p}\big)^3}$$

  the sign of the derivative is the sign of $2Fx+R[(1-\tilde{p})^2-x\tilde{p}(1+\tilde{p})]$ which is positive (using that $\tilde{p}<F/R$).

The effect of x on the regulator surplus at the optimal scheme, by an envelop argument, it is

$$\frac{\partial V}{\partial s_1}(x\sigma_2,\sigma_2)\sigma_2$$

which is negative.

Welfare is decreasing with respect to $\tilde{p}$ as long as $\tilde{p}>p^{FB}$.

B Proof of Proposition 2

- First step: $s_2^*=0$:

The regulator maximizes its surplus (eq. 14) subject to the non-negativity constraints on profit (eq.15) and subsidy s_1 and s_2. The Lagrangien is:

$$\mathcal{L}=v(\theta,e(s_1-s_2),s_1,s_2)+\mu_0\pi+\mu_1 s_1+\mu_2 s_2$$

With μ_0 the Lagrange mulltiplier associated to the participation constraint, μ_i the maultiplier associated with the non-negativity constraint of s_i, $i=1,2$. At the optimum

$$p_e(e,\theta)[b-(s_1-s_2)]e'-(1-\mu_0)p+\mu_1=0 \tag{30}$$

$$-p_e[b-(s_1-s_2)]e'-(1-\mu_0)(1-p)+\mu_2=0 \tag{31}$$

And the corresponding slackness conditions. There are eight possible situations depending on whether each Lagrange multiplier is null or positive. Summing the two equations gives

$$\mu_1+\mu_2+\mu_0-1=0 \tag{32}$$

Let us denote s_1^* and s_2^* the optimal subsidies.

- At least one of the μ_i is positive: otherwise $\mu_1 = \mu_2 = 0$ then $\mu_0 = 1$ and $s_1^* - s_2^* = b$. Welfare is then $-(1-p)s_2^* < 0$, which cannot be optimal.

  Consequently, $\mu_1 + \mu_2 > 0$ and $\mu_0 < 1$ (from eq. (32)).

- We show by contradiction that $b > s_1^* - s_2^*$: otherwise, from equation (30) $\mu_1 = (1-\mu_0)p - p_e[b - (s_1^* - s_2^*)]e' > 0$, which implies that $s_1^* = 0$ and $s_2^* < s_1^* - b = -b < 0$ a contradiction.

- Then $s_2^* = 0$: from equation (31): $\mu_2 = (1-\mu_0)(1-p) + p_e[b - (s_1^* - s_2^*)]e' > 0$.

• Second step: Expressions of the optimal subsidy

There are four possible cases: i) $s_1^* = 0$ and $\pi > 0$, ii) $s_1^* = 0$ and $\pi = 0$, iii) $s_1^* > 0$ and $\pi > 0$, or, iv) $s_1^* > 0$ and $\pi = 0$.

Case i) corresponds to "business as usual" no subsidy is used and the project is implemented with suboptimal effort. Case ii) corresponds to a situation in which the project is not profitable and it is not worth subsidizing it.

In case iii) $s_1^* > 0$ and $\pi > 0$ then $\partial p/\partial e[b - s_1^*]e' = p$, and in case iv) $s_1^* > 0$ and $\pi = 0$ then $p_e[b - s_1^*]e' - p = -\mu_0 p \leq 0$.

The subsidy $s_{1A}(\theta)$ defined by equation (21) is the solution of $\partial p/\partial e[b - s_1^*]e' = p$. And $s_{1B}(\theta)$ is the solution of $\pi(\theta, e(s_1), s_1, 0) = 0$, replacing e by $(R + s_1)/\gamma$ in eq. (15) gives a second order equation in $(R + s_1)$ with one positive root given by equation (22).

If $s_1^* > 0$ and $\pi > 0$ then $s_1^* = s_{1A}$, and if $s_1^* > 0$ and $\pi = 0$ then $s_1^* = s_{1B}$. Furthermore, if both expressions s_{1A} and s_{1B} are positive the optimal subsidy is the larger of the two.

•Third step: Definition of the thresholds

θ^{BAU} is the solution of $s_{1B}(\theta) = 0$, it is the lowest θ at which a null subsidy ensures deployment. θ_A is the solution of $s_{1A}(\theta) = \max\{s_{1B}(\theta), 0\}$.

- From the the expressions (21) and(22) $s_{1A} > s_{1B}$ if and only if

$$\frac{(b+R)^2}{4} > \left(\frac{\gamma\theta}{1-\theta}\right)^2 + 2F\frac{\gamma}{1-\theta}$$

the right hand side being strictly increasing with respect to θ and, converging toward infinity as θ approaches 1, so that $\theta_A < 1$ and $s_{1A} > s_{1B} \Leftrightarrow \theta < \theta_A$.

- $\theta_A \geq 0$ if and only if $R + b < 2\sqrt{2F\gamma}$, otherwise, if $R + b > 2\sqrt{2F\gamma}$, then for all $\theta \in [0, 1]$ $s_{1B} > s_{1A}$, that is $\pi(\theta, e(s_{1A}), s_{1A}, 0) < 0$.
- $s_{1B}(\theta^{FB}) = b$ (if $\theta^{FB} > 0$) by definition of θ^{FB}: $p(e^{FB}, \theta^{FB})(R + b) - F - f(e^{FB}, \theta^{FB}) = 0$, so $0 = v(\theta^{FB}, e^{FB}, b, 0) + \pi(\theta^{FB}, e(b), b, 0) = \pi(\theta^{FB}, e(b), b, 0)$.

We can now look at the optimal solution

- Case 1: $R + b < 2\sqrt{2F\gamma}$:
  - Then $s_{1A} < s_{1B}$ for all θ, which implies that $p_e[b - s_1]e' < p$ for all $s_1 \geq s_{1B}$, for all θ. The optimal subsidy is then either $s_1^* = 0$ or $s_1^* = s_{1B}$.
  - The surplus of the regulator for $s_1 = s_{1B}(\theta)$ is $v = p(e(s_{1B}(\theta)), \theta)(b - s_{1B}(\theta))$, it is positive if and only if $b > s_{1B}(\theta)$, that is, $\theta > \theta^{FB}$.
  - For $\theta > \theta^{BAU}$: the project is implemented and no surplus is created by a marginal increase of effort ($p_e b - p < 0$) so $s_1^* = 0$.
- Case 2: $R + b \geq 2\sqrt{2F\gamma}$:
  - For $0 < \theta < \theta_A$, $s_{1A} > s_{1B}$ so that $\pi(\theta, e(s_{1A}), s_{1A}, 0) > 0$, and $p_e[b - s_1]e' - p > 0$ for both $s_1 = s_{1B}$ and $s_1 = 0$, so that $s_1^* = s_{1A}$.
  - For $\theta_A \leq \theta < \theta^{BAU}$: $s_{1A} < s_{1B}$ so that $\pi(\theta, e(s_{1A}), s_{1A}, 0) < 0$ and $s_{1B} > 0$ so that $s_1^* = s_{1B}$. This case might not arise if $\theta_A > \theta^{BAU}$ that is $s_{1A}(\theta^{BAU}) > 0$.
  - For $\theta \geq \max\{\theta_A, \theta^{BAU}\}$: the profit is positive for $s_1 = 0$ and $p_e(b - s_1)e' - p < 0$ at $s_1 = 0$ so that $s_1^* = 0$.

C Adverse Selection and Moral Hazard

To alleviate notation the probability $p(e, \theta_L)$ and $p(e, \theta_H)$ are denoted with subscripts: $p_L(e)$ and $p_H(e)$, and the profits π_L and π_H.

Let us denote (s_1^*, s_2^*) the optimal solution. There are four possible types of solution depending on whether each component is positive or null. Lemma (3) derive the expressions of the subsidies when they are positive, Proposition (3) consider the influence of λ on the solution.

C.1 Proof of Lemma 3

If both s_1^* and s_2^* are positive, then low type projects are implemented and their profits are null. The regulator surplus is then

$$v(s_1, s_2) = (1-\lambda)\big[p_L(b-s_1)-(1-p_L)s_2\big]+\lambda\big[p_H(b-s_1)-(1-p_H)s_2\big] \quad (33)$$

and the optimal scheme satisfies the following equation

$$\frac{\partial v}{\partial s_1}\frac{\partial \pi_L}{\partial s_2} - \frac{\partial v}{\partial s_2}\frac{\partial \pi_L}{\partial s_1} = 0$$

that is

$$\frac{\partial v}{\partial s_1}(1-p_L) - \frac{\partial v}{\partial s_2}p_L = 0$$

which gives, denoting $s^* = s_1^* - s_2^*$:

$$\begin{aligned}
\lambda[p_H(1-p_L)+(1-p_H)p_L] &= \Big[(1-\lambda)\frac{\partial p}{\partial e}(e,\theta_L)+\lambda\frac{\partial p}{\partial e}(e,\theta_H)\Big](b-s^*)e' \\
\lambda[p_H-p_L] &= \Big[(1-\lambda)(1-\theta_L)+\lambda(1-\theta_H)\Big](b-s^*)\frac{1}{\gamma} \\
\lambda(\theta_H-\theta_L)(\gamma-(R+s^*)) &= \Big[(1-\lambda)(1-\theta_L)+\lambda(1-\theta_H)\Big](b-s^*)
\end{aligned}$$

which then gives equation (23). The equation (24) corresponds to $\pi_L = 0$.

C.2 proof of Proposition 3

The solution $s_1^* = s_2^* = 0$ corresponds to the situation in which L firms do not enter. The regulator surplus in that situation is:

$$V_1(\lambda) = \lambda p_H b$$

In all other situations, if one of the optimal subsidy is positive, L firms do enter (from Proposition 2, if only H firms enter then it is optimal to set $s_1 = s_2 = 0$). The regulator surplus when L firms enter is

$$V_2 = (1-\lambda)[p_L(b-s_1) - (1-p_L)s_2] + \lambda[p_H(b-s_1) - (1-p_H)s_2]$$

that can be equivalently defined as a function of $s = s_1 - s_2$ and s_2:

$$V_2(\lambda, s, s_2) = (1-\lambda)[p_L(b-s) - s_2] + \lambda[p_H(b-s) - s_2]$$

and the constraint $s_1 \geq 0$ is then $s + s_2 \geq 0$.

The problem of the regulator can be decomposed in two steps: first maximize V_2 and then compare the maximum obtained with V_1.

Let us consider the maximization of V_2 subject to $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ and denote $s^{**}(\lambda)$ and $s_2^{**}(\lambda)$ the solution, and $s_1^{**} = s^{**} + s_2^{**}$. The problem can be simplified by transforming the three constraints $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ into two constraints on s, by parameterizing everything by s.

• At the maximum $\pi_L = 0$: by contradiction, if $\pi_L > 0$ then $s_2^{**} = 0$ and s_1^{**} is larger than s_{1B} (which cancels π_L, it is defined by eq. 22) and solves

$$[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda \frac{\partial p_H}{\partial e}](b - s_1)e' = (1-\lambda)p_L + \lambda p_H$$

then $\partial p_L / \partial e (b - s_1^{**})e' > p_L$ that is $s_1^{**} < s_{1A}(\theta)$ (given by eq. 21) which is lower than $s_{1B}(\theta)$ when $(R+b) \geq 2\sqrt{2F\gamma}$ (proof of Proposition 2), a contradiction.

• We can then define $s_2(s)$:

$$s_2(s) = F - \max_e [p(e, \theta_L)(R+s) - f(e, \theta_L)]$$

it is decreasing with respect to s with $s_2'(s) = -p_L$. And $s_1(s) = s + s_2(s)$ is strictly increasing with respect to s ($s_1' = 1 - p$).
- For $s = -R$, $s_2(-R) = F$ and the associated s_1 is $F - R < 0$.
- At $s = s_{1B}$, the profit $\pi_L(e, s_{1B}, 0)$ is null so that $s_2(s_{1B}) = 0$, and $s > s_{1B} \Leftrightarrow s_2(s) < 0$. Note also that $s_{1B} < b$.
- At $s = 0$, $s_2(0)$ is positive equal to $-\pi_L(e, 0, 0)$.
- Define $\underline{s}$ the solution of $s + s_2(s) = 0$, it is between $-R$ and 0. The corresponding s_2 is such that $\pi_L(e, 0, s_2) = 0$.

The regulator's objective is then equivalent to the maximization of

$$\max_s V_2(\lambda, s, s_2(s)) \text{ s.t. } \underline{s} \leq s \leq s_{1B}$$

The derivative of the objective function with respect to s is:

$$\begin{aligned}
\mathcal{V}(\lambda, s) &= \Big[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda\frac{\partial p_H}{\partial e}\Big](b-s)\frac{1}{\gamma} - \lambda[p_H - p_L] \\
=&\Big[(1-\lambda)(1-\theta_L) + \lambda(1-\theta_H)\Big](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - \frac{R+s}{\gamma}) \\
=&\Big[(1-\theta_L) - 2\lambda(\theta_H - \theta_L)\Big](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - e^{FB}) \text{ using 17} \\
=&(\theta_H - \theta_L)\Big[(\underline{\lambda} - \lambda)(b-s)/\gamma - \lambda(1 - e^{FB})\Big]
\end{aligned}$$

in which

$$\underline{\lambda} = \frac{1-\theta_L}{2(\theta_H - \theta_L)}$$

This derivative is strictly decreasing with respect to s as long as $\lambda < \underline{\lambda}$. It is also decreasing with respect to λ for $s < s_{1B}$.

For all $s \in [\underline{s}, s_{1B}]$ we have $\mathcal{V}(0, s) = (1-\theta_L)(b-s)/\gamma > 0$ and $\mathcal{V}(\underline{\lambda}, s) < 0$.

So we already know that $s^{**}(0) = s_1^{**}(0) = s_{1B}$ and $s_2^{**}(0) = 0$, and that, $\forall \lambda > \underline{\lambda}$, $s^{**}(\lambda) = \underline{s}$: $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s})$ the solution of

$$p_L(e)R + (1-p_L)s_2 = F + f_L(e)$$

And we can define :

- λ_1 the solution of $\mathcal{V}(\lambda, s_{1B}) = 0$
- λ_2 the solution of $\mathcal{V}(\lambda, \underline{s}) = 0$

Then the optimal solution as a function of λ is such that

- $0 \leq \lambda < \lambda_1$: $s^{**}(\lambda) = s_1^{**}(\lambda) = s_{1B}$ and $s_2^{**}(\lambda) = 0$
- $\lambda_1 \leq \lambda < \lambda_2$: $s^{**}(\lambda) \in (\underline{s}, s_{1B})$, $s_1^{**}(\lambda) > 0$ and $s_2^{**}(\lambda) > 0$
- $\lambda_2 \leq \lambda \leq 1$: $s^{**}(\lambda) = \underline{s}$, $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s}) > 0$

Then, the regulator should compare V_2 and V_1, the difference $V_2 - V_1$ is decreasing with respect to λ and positive for $\lambda = 0$ and negative for $\lambda = 1$ (by Proposition 2). There is then a λ_3 so that $\lambda > \lambda_3$ implies $s_1^{**}(\lambda) = s_2^{**}(\lambda) = 0$.

C.3 Proof of Proposition 4

The proof and the exposition are easier if we work with $s = s_1 - s_2$ and s_2. That is, the regulator proposes a menu $\{(s_L, s_{2L}), (s_H, s_{2H})\}$, and if a firm of type i chooses the item (s_j, s_{2j}) with $i, j \in \{H, L\}$ it exerts the effort $e(s_j)$ and gets

$$\pi_{ij} = \theta_i(R + s_j) + (1 - \theta_i)\frac{(R + s_j)^2}{2\gamma} - F + s_{2j}$$

and to further alleviate the exposition we denote $p_i = p(\theta_i, e(s_i))$ and $p_{HL} = p(\theta_H, e(s_L))$.

• At the optimal menu m^* both types deployed their project:

If the low type project is deployed so is the high type project. The agency can propose a menu m_0 with $(s_L, s_{2L}) = (-R, F)$ and $(s_H, s_{HL}) = (b, F - \theta_H(R + b) + (1 - \theta_H)(R + b)^2/(2\gamma))$, with this menu the agency extracts the maximum surplus from high types and gets a surplus from low types. It cannot do better by discouraging low type and only subsidizing the deployment of high types.

• The optimal menu m^* is then the solution of:

$$\max_m \lambda[p_H(b - s_H) - s_{2H}] + (1 - \lambda)[p_L(b - s_L) - s_{2L}] \tag{34}$$

subject to $\pi_{ii} \geq 0$ for $i = H, L$, and $\pi_{ii} \geq \pi_{ij}$ for $i, j \in \{H, L\}$.

Form the first step, the optimal menu is m_0 with $s_L^* = -R$ (the low type exerts no effort and gets zero profit, the agency can then extract the maximum surplus from a high type) or such that $s_L^* > -R$. Any menu with $s_L < R$ cannot generate more surplus than m_0.

Then, the only two binding constraints at the optimal menu are $\pi_{LL} \geq 0$ and $\pi_{HH} \geq \pi_{HL}$ because $R + s_L \geq 0$, so that $\pi_{HL} > \pi_{LL}$ and at the optimal scheme $s_H^* > s_L^*$ (to be checked at the end) so that $(\pi_{HH} - \pi_{HL}) - (\pi_{LH} - \pi_{LL}) = (\theta_H - \theta_L)(s_H^* - s_L^*)$.

Then, if $s_L^* > -R$ a low type firm exerts an effort, write the Lagrangien

$$\begin{aligned}\mathcal{L}(s_L, s_{2L}, s_H, s_{2H}) = \lambda[p_H(b - s_H) - s_{2H}] &+ (1 - \lambda)[p_L(b - s_L) - s_{2L}] \\ &+ \mu_L \pi_{LL} + \mu_H(\pi_{HH} - \pi_{HL}]\end{aligned}$$

with μ_L and μ_H the Lagrange multipliers of the constraints $\pi_{LL} \geq 0$ and $\pi_{HH} \geq \pi_{HL}$ respectively. Then the optimal menu satisfies the KKT conditions:

$$\frac{\partial \mathcal{L}}{\partial s_H} = \lambda \frac{\partial p_H}{\partial e}(b - s_H)e' + (\mu_H - \lambda)p_H = 0 \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial s_{2H}} = \mu_H - \lambda = 0 \tag{36}$$

$$\frac{\partial \mathcal{L}}{\partial s_L} = (1-\lambda)\frac{\partial p_L}{\partial e}(b - s_L)e' + (\mu_L - (1-\lambda))p_L - \mu_H p_{HL} = 0 \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial s_{2L}} = -(1-\lambda) + \mu_L - \mu_H = 0 \tag{38}$$

Form eq. (35) and (36) $s_H^* = b$. From eq. (36) and (38) $\mu_L = 1$, and together with eq. (36) it gives

$$(1-\lambda)(1-\theta_L)(b - s_L)\frac{1}{\gamma} = \lambda(p_{HL} - p_L) = \lambda(\theta_H - \theta_L)(1 - \frac{R + s_L}{\gamma})$$

which, after some manipulations gives the expression (25) for s_L^* in Proposition 4. The optimal subsidy s_{2L}^* cancels a low type profit, and the subsidy s_{2H} is found with the constraint $\pi_{HH} = \pi_{HL}$.

If the above scheme can be implemented it outperformed m^0. The subsidy s_L^* is decreasing with λ, and λ_{menu} is the solution of $s_L^* = -R$ (with the expression 25). Note that the denominator in (25) is positive for $\lambda \in [0, \lambda_{menu}]$.

D Proof of Corollary 9

Proof.

From Proposition 1, equation (8) the threshold value of b is

$$\frac{R^3}{F(R-F)}\frac{1}{2}(1 - p^{BAU})^2 = \frac{R(R-F)}{2F}$$

then if b is larger than this value the optimal threshold type p^{SB} solves equation (9):

$$p^{SB} = p^{FB} + \frac{R-F}{R+b}\int_{p^{SB}}^{1} \frac{1-p}{(1-p^{SB})^2}dp = \frac{F}{R+b} + \frac{1}{2}\frac{R-F}{R+b} = \frac{1}{2}\frac{R+F}{R+b}$$

and using equation (3) gives (29).

■